influence of scattering on the conversion which can be achieved in a photosensitized reaction occurring in a fluid flowing within the slab.

The mixing state of the reacting fluid is the major point in extending the results of the previous analysis to the chemical reaction since it has been shown [8, 9] that, in the two extreme situations of perfect mixing in the plane normal to the flow direction and of no mixing, conversion depends on the totally absorbed energy (no matter which the $\dot{a}'''$ distribution is) and on the $\dot{a}'''$ distribution respectively.

It can be therefore concluded that scattering will reduce conversion in all those situations where a good radial mixing occurs and therefore conversion is more sensitive to $\dot{\mathscr{E}}''$ than to the $\dot{a}'''$ distribution.

On the contrary scattering can affect positively conversion when, for high values of m, poor radial mixing conditions are considered. In these optical situations, for these flow conditions, the rearrangement of the $\dot{a}'''$ distribution causes the absorbed energy to be more effective for the reaction: this positive effect may overcome the negative effect due to the reduction of $\dot{\mathscr{E}}''$ so that an increase of conversion with respect to the pure absorption situation is the final result. In optically thick media scattering acts therefore as a mixing promoter of the absorbed energy and, in the absence of a satisfactory fluidinamic mixing, an increase of conversion follows consequently from the improvement of the total degree of mixing of the reacting system. Even in these situations, of course, conversion will be improved up to a value of c, depending on m, since in the limit of pure scattering medium ($c \to 1$) no absorption occurs within the slab and no reaction can occur.

As a concluding remark it is worth noting that these results confirm conclusions drawn on the effect of scattering on photosensitized reactions in an annular reactor of finite dimensions [3].

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A SIMPLE ANALYSIS OF UNSTEADY HEAT TRANSFER IN IMPULSIVE FALKNER–SKAN FLOWS

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NOMENCLATURE

a, relaxation parameter defined by equation (19);
f, dimensionless velocity component parallel to the wall {see equation (9a) of [1]};
$f''(0)$, second derivative of f with respect to η evaluated at the wall;
$f_s''(0)$, steady state value of $f''(0)$;
G_n, functions of ξ defined by equation (6b);
H_n, functions of ξ defined by equation (6c);
n, exponent of the coordinate parallel to the wall in inviscid flow relation;
Pr, Prandtl number;
V, dimensionless normal velocity component {see equation (9c) of [1]};
V_n, functions of ξ defined by equation (6a).

Greek symbols

β, wedge angle equal to $2n/(n+1)$;
δ, parameter given as $(1-n)/(1+n)$;
ξ, independent variable defined by equation (5);
η, transformed quasi-similar coordinate {see equation (8b) of [1]};
θ, dimensionless temperature;
ϕ_0, steady state value of $-\partial\theta/\partial\eta$ evaluated at the wall;
τ, dimensionless time {see equations (8a) and (14) of [1]}.

Superscripts

$'$, $''$, to denote first and second derivatives with respect to ξ or η.

INTRODUCTION

In a recent paper, Watkins [1], presented the numerical solution for predicting unsteady heat-transfer coefficient in the case of impulsive Falkner–Skan laminar flows. As clearly explained in that work numerical estimates are not easy to obtain due to the characteristic curve dividing, at all times, Rayleigh and Falkner–Skan boundary-layer regions which must be matched through a transition zone.

The purpose of this short contribution is to show that previous approximate methods developed to predict the rate of heat transfer at unsteady state conditions, but when the flow was steady, (Sparrow and Gregg [2], Cess and Sparrow [3] and Cess [4]) are also of direct application to the present situation.

In a recent analysis [5] the authors have shown that in the case of small Prandtl (Pr) numbers, in the above mentioned works, the convective effect was in actual facts overestimated since the velocity profile was approximated by the leading term of an expansion series from the wall.

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In the present situation the convective effect is even smaller, at short times, since both momentum and thermal boundary layers develop simultaneously. Thus, a very simple solution, at least valid for small Pr, can be obtained after matching Rayleigh and steady state solutions.

ANALYSIS

When $\tau \to 0$, the governing partial differential equations can be written after Watkins [1], as:

$$\frac{\partial V}{\partial \eta} - 2\tau\delta\frac{\partial f'}{\partial \tau} + f' = 0 \qquad (1)$$

$$(2-2\tau\delta f')\frac{\partial f'}{\partial \tau} + \beta(f'^2-1) + V\frac{\partial f'}{\partial \eta} - \frac{\partial^2 f'}{\partial \eta^2} = 0 \qquad (2)$$

$$(2-2\tau\delta f')\frac{\partial \theta}{\partial \tau} + V\frac{\partial \theta}{\partial \eta} - \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} = 0 \qquad (3)$$

with the following initial and boundary conditions:

$$\begin{array}{lll} f' = V = \theta = 0 & \tau = 0 & \eta > 0 \\ f' = V = 0;\ \theta = 1 & \tau \geqslant 0 & \eta = 0 \\ f' = 1;\ \theta = 0 & \tau \geqslant 0 & \eta \to \infty. \end{array} \qquad (4a,b,c)$$

Equations (1)–(3) are written in dimensionless form representing mass, momentum and energy balances respectively. All variables were defined by Watkins [1] and are also given in the nomenclature.

By introducing a new independent variable:

$$\xi = \eta(2\tau)^{-1/2} \qquad (5)$$

assuming solution series for V, f' and θ of the form:

$$\begin{array}{l} V = V_1(\xi)\tau^{1/2} + V_2(\xi)\tau^{3/2} + \ldots \\ f' = G_0(\xi) + G_1(\xi)\tau + \ldots \\ \theta = H_0(\xi) + H_1(\xi)\tau + \ldots \end{array} \qquad (6a,b,c)$$

which after replaced in equations (1) to (3) and equating terms with like powers of generate the following system of linear ordinary differential equations:

$$V_1' + \sqrt{2}\delta\xi G_0' + \sqrt{2}G_0 = 0 \qquad (7)$$

$$2\xi G_0' + G_0'' = 0 \qquad (8)$$

$$2\xi G_1' + G_1'' - 4G_1 = 2\delta G_0 G_0'\xi + \beta(G_0^2 - 1) + \sqrt{2}V_1 G_0' \qquad (9)$$

$$2\xi H_0' + Pr^{-1}H_0'' = 0 \qquad (10)$$

$$2\xi H_1' + Pr^{-1}H_1'' - 4H_1 = 2\delta G_0 H_0'\xi + \sqrt{2}V_1 H_0' \qquad (11)$$

where $'$ and $''$ denote first and second order derivatives with respect to ξ. The following boundary conditions must be met:

$$\begin{array}{ll} V_1 = G_0 = G_1 = H_1 = 0;\ H_0 = 1 & \xi = 0 \\ G_0 = 1;\ G_1 = H_0 = H_1 = 0 & \xi \to \infty \end{array} \qquad (12a,b)$$

Solutions for G_0 and H_0 are straightforward:

$$\begin{array}{l} G_0 = \mathrm{erf}(\xi) \\ H_0 = \mathrm{erfc}(\xi Pr^{1/2}). \end{array} \qquad (13a,b)$$

However the solution for H_1 is not easy to find unless further assumptions are made. In a recent analysis [5] it was shown that, when the flow is steady, second order correction terms are only important, for matching purposes, when Pr is large. Thus, an estimate of this effect can be obtained by assuming:

$$G_0 \simeq (2/\sqrt{\pi})\xi \qquad (14)$$

thus giving:

$$V_1 \simeq -(1+\delta)(2/\pi)^{1/2}\xi^2 \qquad (15)$$

and this in turn allows the calculation of $H_1'(0)$:

$$H_1'(0) \simeq \frac{4(1+\delta)}{30\pi}. \qquad (16)$$

Finally:

$$-\frac{\partial \theta}{\partial \eta}\bigg|_{\eta=0} = \left(\frac{2Pr}{\pi\tau}\right)^{1/2} + \frac{\sqrt{2}(1-\delta)}{15\pi}\tau^{1/2} + 0(\tau^{3/2}) \qquad (17)$$

and since $0 \leqslant \delta \leqslant 1$ it is clearly seen that second order correction is negligible even for Pr to the order of 1 and for values of τ approaching 1.

On the other hand when $\tau \to \infty$ steady state solution is found as was clearly pointed out by Watkins [1]. By neglecting second order terms in equation (17) and following Cess [4] procedure a general approximate solution valid on the whole range of τ values can be constructed, giving:

$$-\frac{\partial \theta}{\partial \eta}\bigg|_{\eta=0} = \left(\frac{2Pr}{\pi\tau}\right)^{1/2} \exp(-a\tau) + \phi_0 \,\mathrm{erf}[(a\tau)^{1/2}] \qquad (18)$$

where:

$$a = (2\phi_0^2/Pr) \qquad (19)$$

ϕ_0 being the steady state value of the dimensionless temperature gradient evaluated at the wall $(\partial\theta/\partial\eta)|_{\eta=0}$.

Incidentally, an approximate expression for $f''(0)$ (where $''$ denotes second order derivative with respect to η) can also be given in a similar fashion:

$$f''(0) = [2/(\pi\tau)]^{1/2}\exp[-2f_s''(0)\tau] + f_s''(0)\,\mathrm{erf}[(2f_s''(0)\tau)^{1/2}] \qquad (20)$$

and once again $f_s''(0)$ denotes the steady state value of $f''(0)$.

Results obtained with equations (18) and (20) are compared in Table 1, with numerical tabulated results presented by Watkins [1]. As can be seen equations (18) and (20) are able to predict within 10% almost all numerical results presented by Watkins [1]. However it should be noted that heat-transfer results are for $Pr = 0.7$. When Pr increases the agreement is not so good. Nevertheless, the worse situation was found for the case $Pr = 10$ and $n = 0$ as shown in Fig. 1. Though the numerical results are fairly well correlated by our equation (18)—since the convective effects were not properly

Table 1. Comparison among numerical tabulated results (N) given by Watkins [1] and approximated results (A) obtained with expressions (18) and (20), $Pr = 0.7$

	$(-\partial\theta/\partial\eta)_{\eta=0}$						$2f''(0)$	
	$n = 1.0$		$n = 0.333$		$n = 0.0$		$n = 0.0$	
τ	N	A	N	A	N	A	N	A
0.1	2.14	2.11	2.13	2.11	2.12	2.11	3.586	3.57
0.5	0.989	1.06	0.961	0.98	0.946	0.95	1.60	1.61
0.8					0.746	0.76		
1.0	0.729	0.71			0.668	0.69	1.125	1.18
2.0	0.582	0.566	0.508	0.54	0.487	0.52	0.805	0.87
4.0	0.510	0.508	0.471	0.49			0.67	0.72
5.0					0.414	0.43		
10.0	0.496	0.496	0.471	0.471			0.664	0.664

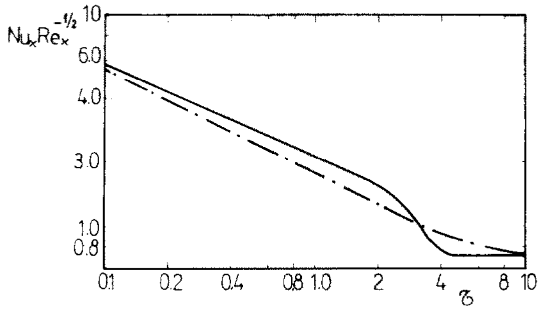

FIG. 1.

taken into account in our analysis neither the time to reach steady nor the form of the curve are properly predicted.

It should finally be mentioned that curves presented by Watkins [1] for the friction factor, are fairly well reproduced by or simple expression (20), when $n \geqslant 0$.

It can be concluded that our simple analysis is able to predict rates of heat and momentum transfer which are in fair agreement with estimates presented by Watkins [1] obtained with a rather difficult numerical technique.

However the more important achievement of this contribution is related with future works since by usual superposition techniques it is possible to analyze other situations where wall temperature is some prescribed time function or either to find wall temperature profile when the flux is prescribed.

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A METHOD OF OBTAINING FLOW FILM BOILING DATA FOR SUBCOOLED WATER

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NOMENCLATURE

- G, mass flux [kg/m² s];
- P, pressure [MPa];
- T_{sat}, saturation temperature [K];
- T_w, heated surface temperature [K];
- X_{in}, inlet quality;
- ΔT_{in}, inlet subcooling [K];
- ϕ, surface heat flux [W/m²].

THE OBJECTIVE of this paper is to describe a simple experimental method to be used for measuring flow film boiling data at low qualities and subcooled conditions.

The most widely used system in forced convective boiling studies is a heat flux controlled system where the heat output of an electrically heated element is increased gradually. Such a system however does not permit the measurement of flow film boiling data for subcooled water since the high CHF values would result in heated surface temperatures in excess of the melting point of most metals. Instead, a variety of temperature controlled systems could be used in film boiling studies. A review of experimental techniques, utilized in post-CHF studies, has been made by Groeneveld [1]. Despite the complexity of some of the techniques used none of these studies has produced fully developed flow film boiling data suitable for deriving a correlation or verifying existing correlations.

The technique used in our study was first discovered accidentally at Chalk River in the late sixties. The technique, as originally used by Groeneveld [2], employed a high thermal inertia hot patch, heated independently and attached to a directly heated tube cooled by Freon (Fig. 1). The hot patch's original purpose was to study the effect of flux spikes on the CHF. However, when the hot patch power was raised such that dryout occurred, the downstream dryout behavior was changed drastically.

The high thermal inertia of the hot patch clamp enabled the experimenter to limit the temperature rise at dryout by reducing the hot patch power. When steady film boiling temperatures were reached at the hot patch the test section power was raised slowly. At power levels well below the dryout power for uniform heating a phenomenon illustrated in Fig. 1 occurred: the dry patch started to spread from the hot patch in the downstream direction. During two runs dry patch spreading was observed visually using an i.r. camera focused on the bare test section tube; the hot region was seen to propagate slowly from the hot patch at an apparently constant rate.

The effect of a hot patch on the downstream thermal behavior of an otherwise uniformly heated tube is shown graphically in Fig. 2. The solid lines represent the boiling curves for uniform heating at constant inlet conditions while the broken lines illustrate the effect of an upstream hot patch in extending the film boiling temperature and heat flux range to lower values. Dryout at the hot patch does not seem to affect the film boiling temperatures as both symbols in Fig. 2 fall along a single curve.

It was realized immediately that this technique of producing film boiling data at heat flux levels well below the CHF could be very advantageous in water at subcooled conditions as here heater failures frequently occurred when the CHF was exceeded accidentally. Initially a similar setup for water was constructed. Because of expected failures of cartridge heaters, the copper hot patch clamp (O.D. 2.5 cm; length 4 cm) was heated by two oxygen-acetylene torches. Although dryout at the hot patch clamp did occur, especially when the flow was reduced, no spreading of the dry patch along the directly